Fair Coloring of Planar Cubic Graphs

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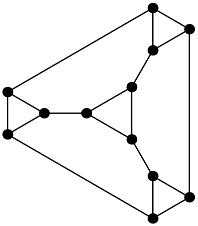
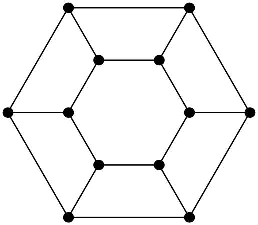
Definition (Planar cubic graph)

We call a graph

- planar if it can be drawn in the plane without crossing, and
- cubic if the degree of all the vertices is three.





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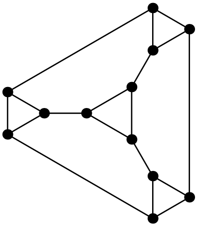

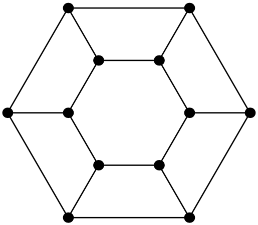

Definition (Coloring)

A *k*-coloring is an assignment of k different colors to the vertices of the graph.

Definition (Proper coloring)

A coloring is proper if no two adjacent vertices have the same color.

Figure: Proper coloring

Figure: Non-proper coloring

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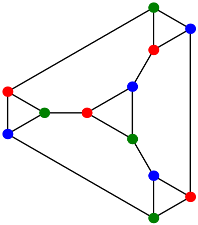

Figure: Proper coloring

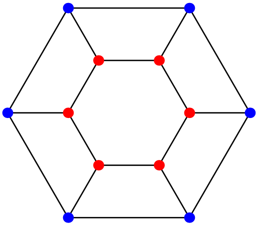

Figure: Non-proper coloring

Theorem (Four color)

Every planar graph has a proper 4-coloring.

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A proper 4-coloring of a cubic planar graph is fair if all the neighbors of each vertex have distinct colors.

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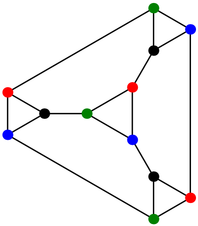

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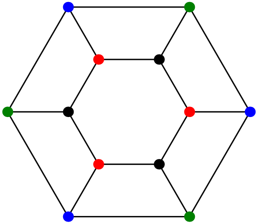

Figure: Non-fair coloring

Problem

For a given cubic planar graph we want to decide if there exists a fair coloring of the graph.

Question

How difficult is such problem? Does there exist an efficient algorithm?

- Fair coloring of cubic graphs is hard.
- Fair-like coloring of subcubic planar graphs is also hard.

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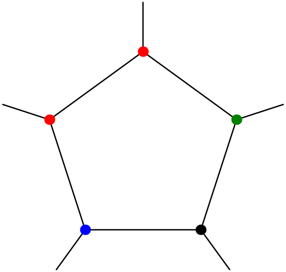

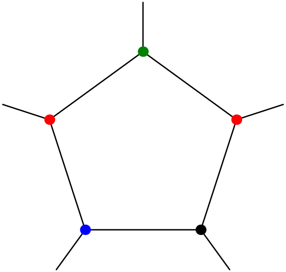

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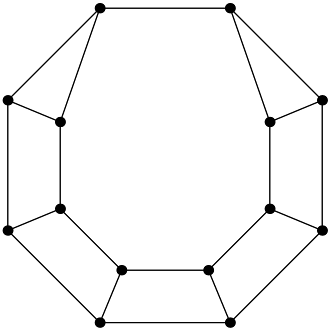

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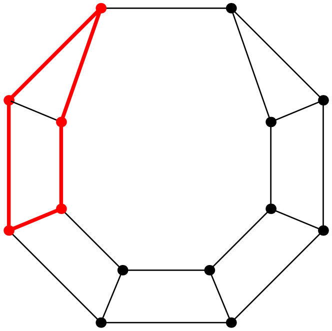

Any questions or suggestions?