3- choosability of plane graphs

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Planar graph

Is planar  Is planar  Isn't planar
Coloring of a graph

- **Coloring**
  
  \[ col : V \rightarrow \{1...n\} \]

- **Proper coloring**
  
  \[ \forall uv \in E : col(u) \neq col(v) \]

- **Chromatic number**
  
  \[ \chi(G) = \min \{ \exists \text{ proper coloring } col : V \rightarrow \{1..n\} \} \]
List coloring

- Every vertex has list of possible colors
- Vertex may get color only from its list
- Proper list coloring – no same color on adjacent vertices
- Choosability

\[
\chi_l(G) \quad \min_n \{ \forall \text{ lists } l \forall v \in V : |l(v)| \geq n \exists \text{ proper coloring} \}
\]
Choosability vs. Coloring

∀ \( G : \chi(G) \leq \chi_1(G) \)

∃ \( G : \chi(G) \neq \chi_1(G) \)

\( \chi(G) = 2 \)

\( \chi_1(G) > 2 \)
Our Task, What is known

- Every planar bipartite graph is 3-choosable
- Not every planar graph without triangle is 3-choosable

- Try if some locally bipartite graphs are 3-choosable.